Colosseum Calculation Assumptions

Demand

1) The underground passageways below the arena were not built during the initial phase of construction. The foundation below the ring area of the Colosseum was solid (e.g. did not have passageways built underneath it), and was made of concrete with a three metre-thick brick retaining wall on both the inner and outer edges of the ring.

2) Concrete was weight-bearing, and was used in all vaults, and in the floors and walls above level 1. Concrete was composed of water, lime, and pozzolano, in a respective ratio of 0.175 : 0.275 : 0.55. This corresponds to an average of 15-20% water, and one part lime to one part pozzolano (based on Adam: p. 74, who cites Vitruvius V, XII, 8-9, Maritime Works). The wet and dry mass were assumed to be the same (i.e. the water chemically bonds to the lime/pozzolano in the drying process rather than evaporating).

3) The following densities of materials were used for calculations:

a.	Travertine	2,720 kg/cu m
b.	Tufa	2,225 kg/cu m
c.	Brick	2,403 kg/cu m
d.	Pozzolano	1,602 kg/cu m
e.	Lime	849 kg/cu m
f.	Water	1,000 kg/cu m
g.	Rubble	2,243 kg/cu m
h.	Earth	1,442 kg/cu m

4) Materials were transported over the following distances:

   a. Travertine: transported from the quarries at Tivoli (Tibur) to Rome, approximately 30 km.
   b. Tufa: abundant throughout Italy, and probably transported about 1 km.
   c. Brick: Transported from within a range of 1 km.
   d. Pozzolano: available within 3 km of Rome
   e. Lime: transported from the vicinity of Tivoli, 30 km assumed
   f. Water: Transported 300 m
   g. Rubble: much concrete would probably have been recovered from Nero's destroyed complex. We have assumed a 1 km transport range for concrete.
   h. Marble: Would have been transported over water for the bulk of its journey. Work involved in transport over water not included in calculations. Assume 1 km transport over land, to Rome, from port.
   i. Earth: would have to be moved 500 m from site of excavation

Colosseum Calculation Assumptions *continued...*

5) Horizontal movement is calculated using the following formula to calculate work:

   Work (in joules) = (Frictional coefficient) x (gravitational constant) x (mass) x (distance)

   This formula calculates work for a mass being pulled or pushed continuously across a flat, level surface (e.g. no hills, no stopping and starting). In fact, the materials being transported would have been on carts with wheels, which would likely make the work easier. To account for this, we have chosen a low frictional coefficient (0.1). We recognize that the area between Tivoli and Rome is not devoid of hills, but assume that inclines and declines in the geography roughly cancel one another out.

6) We have assumed that all the horizontal movement was done by oxen, with two oxen per cart. This assumption is based on the understanding that Roman yoke technology was not far advanced, and that four-team yokes were therefore not well developed. We have assumed the journey to the Colosseum would have taken twice as long as the return journey to the quarry or production site, and that the Tivoli to Rome would take the oxen 2 days to traverse with a heavy load. As such, in order to have a steady stream of materials flowing daily to the construction site, there would have to have been 3 teams of oxen. For every two teams of oxen bringing materials to the Colosseum, there would have been one team making the return journey. We have assumed one driver per cart, meaning that the number drivers would have been equal to the number of oxen teams. We have assumed a constant rate of movement for the oxen.

7) In order to approximate the work required to move materials around the construction site we have assumed there were 4 equidistant drop off points for materials coming in from outside. We have further assumed that materials were dropped at the drop point closest to the area where the work was to be done. Using this rationale, no onsite material would have to be moved more than one-quarter of the circumference of the outer ellipse of the Colosseum. Further, no onsite material would have to be moved more than one-half the radial depth of the Colosseum. We have used a higher frictional coefficient for (0.3) for horizontal movement around the Colosseum, assuming that much of the material would have been moved by humans, likely using logs as rolling devices, rather than pulled by ox cart.

8) We have included the work required to:

   a. Excavate the earth for the foundation
   b. Transport the materials (calculated on the basis of the formula in point 5 above)
   c. Hoist the materials into place. Calculated using the equation: mass x gravitational constant x height.
   d. Move the materials around on-site (based on 5 and 7 above)
   e. Quarry the travertine and tufa, based on DeLaine
   f. Produce the bricks
   g. Quarry/excavate the rubble
   h. Excavate the lime
   i. Slake the lime
   j. Quarry the pozzolano
   k. Shore the foundations
   l. Lay the foundations

Colosseum Calculation Assumptions *continued...*

m. Lay the brick and core for brick-faced walls; lay the brick for the floors of levels 2,3 and 4
n. Mix the mortar
o. Erect scaffolding
p. Prepare and erect centering
q. Load various materials into baskets
r. Lay vaults
s. Jimmy and adjust stonework, check for level and plumb

Items a-d above were calculated using the physics equations outlined earlier, while items e-s were calculated on the basis of figures given by DeLaine. These figures are outlined in the table at the end of this document. Where DeLaine gives labour constant equations broken down by unskilled, skilled and supervisory, we have combined the three equations into one.

9) We have assumed that all the bricks used were half-bessales, except in the case of the floors, where it has been assumed that full bessales were used. The dimensions of the bessale were based on the lydium size of brick cited by Adam (pp. 61-62) and the tubuli size of brick cited by DeLaine (p. 116). The dimensions are as follows:

   a. Height: 9 cm
   b. Depth: 14.80 cm
   c. Length: 29.60 cm

   We have further assumed that there was 1 cm of mortar laid between each brick, and that the length-wise side of the brick would be the side exposed.

10) We have not included work for the following activities:

   a. Design structure
   b. Measure and lay guidelines for site and structures (dimensions, angles, checking for level)
   c. Harvest and transport lumber; prepare scaffolding and centering.
   d. Load and unload materials from carts.
   e. Assemble, disassemble, and move cranes around the site.
   f. Forge and build metal structures (pipes, iron connecting joints for stones).
   g. Plan, administer and manage the building project.
   h. Support the workers, for instance the work required to house, clothe and prepare food for them.

11) We have assumed an efficiency rate of 50% for human work. The human energy requirement before BMR and thermal loss was therefore doubled in our calculations, to account for inefficiency. No efficiency modifications were made to the energy requirements of oxen or for workers driving oxen, for work accounted for by the DeLaine figures, or for basal metabolic calories.

Colosseum Calculation Assumptions *continued...*

12) We have made the following assumptions about the building dimensions, in the interest of simplicity:

    a. Calculations are based on eighty identical segments or “spokes” of the Colosseum. This is not technically accurate, as there were special entranceways at the intersection of the major and minor axes and the façade that were different from the typical segment.
    b. All arches were semi-circular, and therefore, the rise is equal to ½ the span in all instances. This is not accurate, but immensely simplified the calculations, as equations for a simple circle could be used in all instances, rather than having to calculate arc lengths and areas and elliptical segments.
    c. All vaults – including annular, radial, and diagonal vaults – were barrel vaults. We recognize that this is not accurate, as both cross-vaults and “conical” vaults were used in construction.
    d. Stairways were not included in the calculation. This assumption was made on the basis that the ceiling area above any stairway would have been open space. We have assumed that the material required for a stairway would be roughly equal to the ceiling mass had the stairway not been built. The ceiling mass and stairway mass are thus assumed to cancel one another out. We recognize that there were circumferential staircases on the upper level, but have omitted these from our calculations.
    e. Doorways leading from level one into the arena, and from levels 2 and 3 into the respective seating tiers were not calculated.
    f. We have not included columns, statues, or fountains in our calculations – with the exception of the columns on the inner ellipse of the fourth level.
    g. Because we did not have a complete set of measurements for all elements of the Colosseum, some interpolation/extrapolation was required. Based on calculations of the difference between the façade wall ellipse and the arena wall ellipse, we have determined that the width of a segment wedge decreased by 8 cm for every metre distance from the façade wall. We have assumed that the cm decrease in width is equally distributed amongst all elements at a given distance from the perimeter (e.g. the cm decrease is distributed equally across the width of a walkway and the width of the piers; both the walkway and pier widths change).

13) We have used the standard figure of 6000 tons of marble, and assumed that it was used in the first tier of seating only.

14) We have assumed that the basal metabolic requirement of each worker was 1694.03 kcal per day, and that each worker could expend a maximum of 1321.34 surplus calories per day. The available surplus calories were determined using a Physical Activity Level (PAL) factor of 1.78, which we have deemed appropriate based on our research. We have assumed that the average weight of a worker was 70 kg. The BMR was determined using the equation:

    Basal metabolic caloric requirement (kcal) = 70*(weight of worker (kg))^0.75

15) We have assumed that only a portion of the 1321 surplus human calories could be converted into work, as a good deal of this energy would be lost as heat. We have assumed that 40% of surplus calories could have been translated into work, with the remainder lost as heat.

Colosseum Calculation Assumptions *continued...*

16) We have assumed that work would have occurred on 220 out of 365 days. We have included the calories required to support these workers in their off days, using a PAL of 1.55.

17) We have assumed that the basal metabolic requirement of each ox was 6261 kcal per day, and that a maximum of 4883.6 surplus calories could be expended by each ox per day. The available surplus calories were determined using a Physical Activity Level (PAL) factor of 1.78, which we have deemed appropriate based on our research. We have assumed that the average weight of an ox was 400 kg. The BMR was determined using the equation:

    Basal metabolic caloric requirement (kcal) = 70*(weight of ox (kg))^0.75

18) We have assumed that only a portion of the 4883.6 surplus oxen calories could be converted into work, as a good deal of this energy would be lost as heat. We have assumed that 40% of surplus calories could have been translated into work, with the remainder lost as heat.

19) We have assumed that work would have occurred on 220 out of 365 days. We have included the calories required to support these oxen in their off days, using a PAL of 1.55.

20) The BMR was determined using the same equation for humans and oxen. This has been deemed appropriate based on the literature, which indicates that mammalian BMR is linearly related to body weight regardless of species. Based on this logic, we have assumed that the PAL factors for humans and oxen are also equivalent.

21) We have assumed that the conversion efficiency of muscle is the same regardless of species – both humans and oxen are assumed to be able to convert 40% of available surplus energy into work, with the remainder being lost as heat.

22) We have assumed a work year of 220 days for urban construction workers.

Supply

23) For human food, we have assumed that 100% of the caloric content of food listed by the USDA is available for work. For oxen fodder, we have used the figure for digestible energy of dry matter alfalfa, which takes into account the bovine capacity to digest cellulose.

24) Based on our research, we have assumed that the energy content of contemporary grains is similar to that of Roman grains.

25) Based on our research, we have assumed that 30% of food would have been lost to rot and contamination by vermin, while 10% would have been stored as seed for sowing the following year.

26) We have assumed a work year of 300 days for rural agricultural workers.

Colosseum Calculation Assumptions *continued...*

Wheat

27) We have assumed that the only food eaten by workers was hard white wheat. We understand that fruit, vegetables, legumes, wine and oil would also have been eaten, but have omitted these from our calculations in the interest of simplicity.

28) We have assumed that wheat was sown at the rate of one peck to the iugerum, which translates into approximately 145 litres per hectare.

29) We have assumed a yield of 10:1 for sown wheat, such that 1 litre of wheat seed would yield 10 litres of wheat, which translates into 1,157.94 kg of wheat per hectare.

30) Based on our research, we have concluded that 14.25 labour days were required to farm one iugerum of wheat. This translates into roughly 57 labour days per hectare. We have assumed that one labour day is equal to 3026 kcal, so that 173,500 kcal were required to farm 1 hectare of wheat. We have subtracted this number from the total caloric yield of a hectare of wheat, in order to ascertain the caloric surplus available for feeding Colosseum workers.

31) We have assumed that in any given year, half of the land available for wheat cropping would have been fallow.

32) We have based our calculations on a density of wheat equal to 800 kg / cu m, and an energy content of wheat equal to 3,420 kcal / kg.

Hay

33) We have assumed that the only food consumed by Colosseum oxen was hay, made from alfalfa. It is our understanding that alfalfa (then called lucerne) was cultivated for feeding animals.

34) We have assumed that the annual yield of hectare of alfalfa is 15 metric tons fresh matter, which yields 2.6 metric tons of dry matter (based on 82.7% moisture content). Thus one hectare of alfalfa hay yields 2,600 kg of hay.

35) We have assumed that in any given year, half of the land available for hay cropping would have been fallow.

36) Based on our research, we have concluded that approximately 40 human labour days per hectare were required to farm hay. We have assumed that one human labour day is equal to 3026 kcal, so that roughly 120,000 kcal were required to farm 1 hectare of hay. We have subtracted this number from the total caloric yield of a hectare of hay, in order to ascertain the caloric surplus available for feeding Colosseum oxen.

For detailed references, please see the ***Colosseum Project Walkthrough*** *document which is available at: www.theupsideofdown.com/rome/colosseum/*

Colosseum Calculation Assumptions *continued...*

Table of DeLaine Labour Constants

	Skilled	Unskilled	Supervision	Total
Quarry/produce travertine/tufa				0.887 d/cu m
Quarry/produce brick				5.17 d/1000 pieces
Quarry rubble (quarry pumice)				0.375 d/cu m
Produce lime				4.07 d/cu m
Quarry pozzolano				0.468 d/cu m
Shore foundations	.015 d/cu m	.015 d/cu m	0.1*skilled	2.1*(0.015 d/cu m)
Slake lime	1.2 d/cu m	...	0.1*unskilled	1.1*(1.2 d/cu m)
Lay foundations	0.35 + 0.01(d-1) d/cu m	...	0.1*unskilled	1.1*(0.35+0.01(d-1) d/cu m)
Lay brick and core for walls/floor	0.5*skilled	0.8hr/100 pieces+0.03(0.8hr/ 100 pieces)(height-1) +0.4/thickness of wall	0.1*skilled	1.6*[0.8 hours per 100 pieces *(0.97+0.03*(height)) +.4/thickness of wall]/12 = # FTEs
Mix mortar	0.7 d/cu m		0.1*unskilled	1.1*(0.7 d/cu m)
Erect scaffolding	2*skilled	0.021d/sq m face	0.1*skilled	3.1*(0.021 d /sq m face)
Prepare and erect centering	0.1 d/sq m	0.1 d/sq m	0.2*skilled	2.2*(0.1 d/sq m)
Load into baskets	0.06 d/cu m	...	0.1*unskilled	1.1*(0.06 d/cu m)
Lay vaults (lay foundation)	0.35 + 0.01(d-1) d/cu m	...	0.1*unskilled	1.1*(0.35+0.01(d-1) d/cu m)
Jimmy/adjust*				(1/6*cu m + 1/60*height)/12

* The Jimmy/adjust figure was invented by the author – it assumes this task took 10 minutes per cubic metre plus one minute for every metre increase in height.